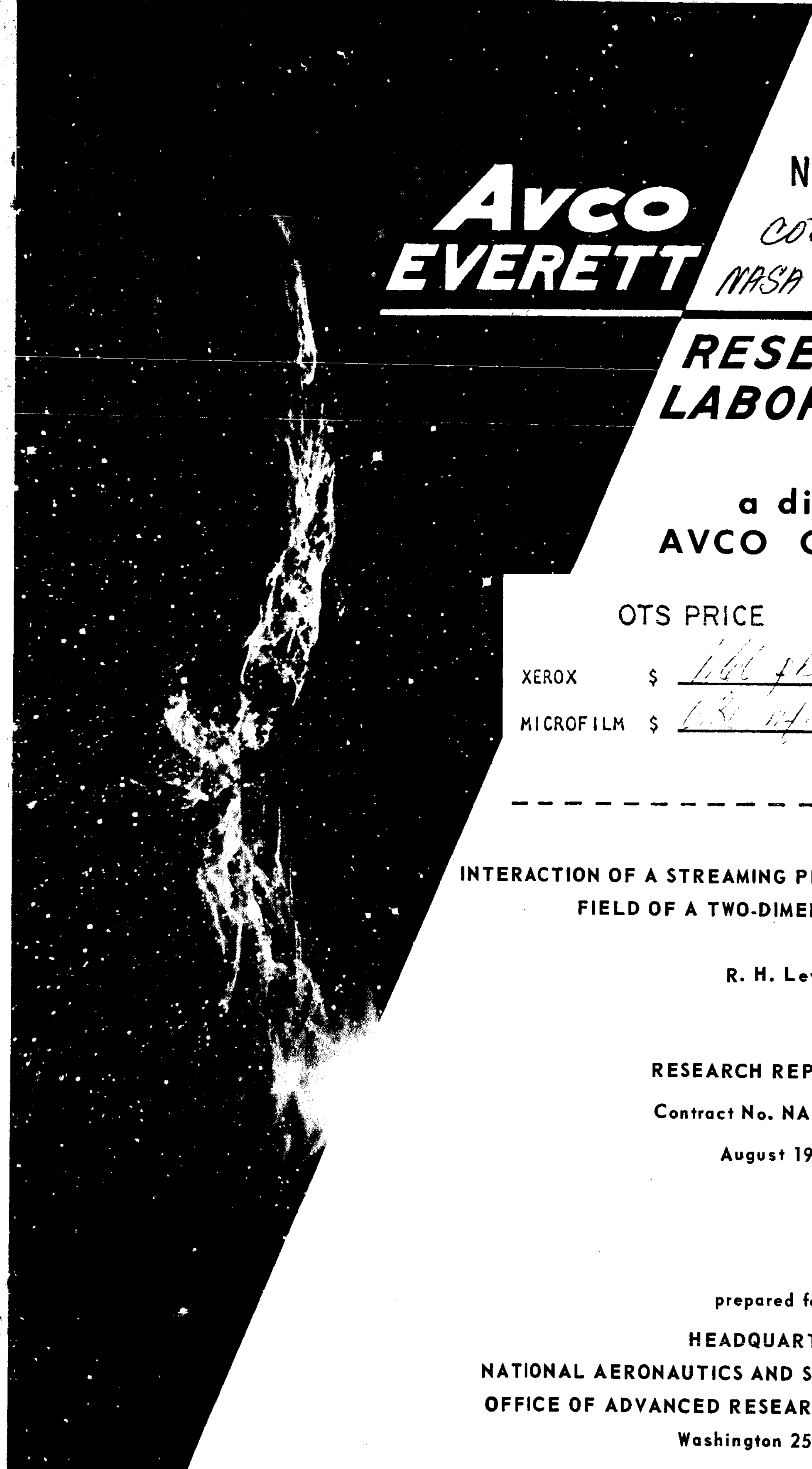

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INTERACTION OF A STREAMING PLASMA WITH THE MAGNETIC FIELD OF A TWO-DIMENSIONAL DIPOLE

R. H. Levy

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ABSTRACT

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The flow of an infinitely conducting plasma past a two-dimensional magnetic dipole oriented parallel to the flow has been considered by Hurley, amongst others. The problem consists of finding a vacuum magnetic field such that along a bounding field line whose location is to be found, the magnetic pressure balances the Newtonian dynamic pressure appropriate to the local slope of the boundary. A related problem has been solved by Cole and Huth; in their case there is no flow, but an isotropic static plasma surrounding the magnetic field region which exerts a constant pressure on the boundary. In the actual flow problem we would expect there to be a stagnant (trapped) region near the front. The stagnant flow would be at nearly constant pressure. Away from this region the Newtonian pressure would again be applicable. This problem, which is a mixture of those cited above, has been solved by an approximate technique due to Cockroft. The solution is shown to have features of both the cited problems, as appropriate.

AUTHOR

INTERACTION OF A STREAMING PLASMA WITH THE MAGNETIC FIELD OF A TWO-DIMENSIONAL DIPOLE

Hurley,[1] Dungey,[2] and Zhigulev and Romishevskii[3] (amongst others) have treated the following problem: a two-dimensional dipole is placed in a uniform hypersonic stream of gas of infinite conductivity, but which carries no magnetic field. A shock wave forms in the gas upstream of the dipole. If the compression ratio across this shock is large there is a thin shock layer just behind the shock in which the pressure is taken to be $\rho_\infty u_\infty^2 \sin^2 \alpha$, where α is the angle between the shock and the oncoming flow. At the back of the shock layer is a current sheet behind which there is a magnetic field but no plasma. Since there is no magnetic field ahead of the current sheet, and since the normal component of the magnetic field is continuous across a current sheet, the trace of the current sheet is a field line. Furthermore, the field strength behind the current sheet is given by:

$$B^2/8\pi = \rho_\infty u_\infty^2 \sin^2 \alpha \tag{1}$$

Behind the current sheet the magnetic field satisfies the usual conditions for such fields in vacuo. The problem is to find the shape of the boundary (and, also the magnetic field within it) consistent with the above condition.

For the case in which the dipole axis is at a substantial angle to the oncoming flow, the solutions referred to above are aerodynamically satisfactory near the stagnation point. However, they imply the existence of a neutral point where $B = \alpha = 0$. Some significance has been attached to the exact position of the neutral point, so it seems appropriate to point out that the centrifugal force in the curved shock layer will reduce the pressure below $\rho_\infty u_\infty^2 \sin^2 \alpha$ at all angles, and to zero for a finite shock angle. (On a cylinder, the pressure goes to zero when $\alpha \approx 35^\circ$). Thus the actual position of the neutral point cannot be determined even approximately using the right hand side of (1) as the aerodynamic pressure.* It is hard to say just what form the boundary might take if the correct pressure (including the centrifugal term) were used, but regions of separated flow are certainly possible. We shall not consider the centrifugal effect further in this paper.

For the case in which the dipole axis is nearly parallel to the flow there are aerodynamic difficulties of a different nature. These difficulties lead one to consider a region of trapped flow. Calculations for these cases using (1) lead to "re-entrant" boundaries to the cavity (see Fig. 1(a)). These shapes are clearly impossible since the fluid between the lines QQ' is deflected inward by the shock; some provision must be made for the escape of this fluid. It seems unlikely that the fluid would escape inwards from the cusp point P in the immediate neighborhood of the dipole. A more reasonable situation is shown schematically in Fig. 1(b). We imagine a region (hatched in Fig. 1(b)) in which there is almost no flow and no magnetic field. The pressure in this region is approximately $\rho_\infty u_\infty^2$. The shock wave joining the points Q can be considered nearly normal; a slight curvature (greater than the distance QQ by an amount roughly equal to the compression ratio across the shock) would allow the gas to escape outwards in a thin layer behind the shock without inducing any substantial flow in the trapped region. The condition on the magnetic field along the lines PQ is that these lines should be field lines, and that the field strength (and hence the magnetic pressure) should be constant along them. Outwards from the points Q where the shock is normal, the usual condition (1) may be applied.

In this note we shall treat the case in which the dipole points directly into the wind. For the calculations that follow we take $\left(8\pi\rho_\infty u_\infty^2\right)^{1/2}$ to be our unit of field, and our unit of distance will be chosen such that the undisturbed dipole produces unit field at unit distance. We shall work in the hodograph plane, and, in order to show how our results may be compared to related problems, we shall treat three problems simultaneously.

Problem (a) is just that of Hurley, the rule (1) being applied along the

*These remarks also apply, of course, to three-dimensional work in this field.

entire shock. Problem (b) is a dipole placed in a stationary isotropic plasma of uniform pressure such that a cavity is formed having constant field strength along its boundary. This problem was solved by Cole and Huth[4]. Our reason for reproducing it here is that it is plainly applicable locally in problem (c), represented by Fig. 1(b). These three problems are shown schematically in Fig. 2. The symmetry of the problem allows us to confine our attention to the region $y \geq 0$. Near the origin in each case we have

$$w \sim -z^{-1}, \; dw/dz = Be^{-i\alpha} \sim z^{-2} \tag{2}$$

where w is the complex potential, $z = x + iy$, B is the magnitude of the magnetic field and α its inclination to the x-axis.

The hodograph plane is defined by

$$\zeta = u + iv = dz/dw = B^{-1} e^{i\alpha} = (\cos\alpha + i \sin\alpha)/B \tag{3}$$

Its utility for the present problems stems from three facts: First, surfaces having unit magnetic pressure map onto sectors of the unit circle. Second, when $v = 1$, $B = \sin\alpha$ which satisfies condition (1) and makes sense whenever $0 \leq \alpha \leq \pi$. It will be observed that this condition on α is satisfied for all relevant portions of the boundaries in Fig. 2. Thirdly, by treating only the upper half of the z-plane, the mapping into the hodograph plane is one-to-one. Figure 3 shows the boundaries appropriate to our three problems in the ζ-plane, with corresponding points labeled with corresponding letters. The real axis in the z-plane, which must be a field line, contains only forward facing ($\alpha = 0$) field vectors and therefore represents a cut along the positive real axis in the ζ-plane. It is easily verified that $w \sim -\zeta^{-1/2}$ near the dipole.

To complete the problems, one further transformation is necessary in cases (a) and (c). We transform to a new plane (the t-plane). For problem (a) we define t by the Schwarz-Christoffel transformation:

$$\pi d\zeta/dt = t/(1-t) \; ; \; \pi\zeta = -t - \ln(1-t) \tag{4}$$

so that the region of interest becomes the upper half of the t-plane; the points NOPR become the points $t = -\infty, 0, 1, \infty$. Near $t = 0$, $\pi\zeta \sim t^2/2$ so that w is given by

$$w = -\sqrt{2\pi}/t \tag{5}$$

Integrating to recover the z-plane,

$$z = \int \zeta dw = \int_0^t \zeta \frac{dw}{dt}\, dt = \zeta w - \int_0^t w \frac{d\zeta}{dt}\, dt = \sqrt{\frac{2}{\pi}}\left[1 + \left(\frac{1-t}{t}\right)\ln(1-t)\right] \tag{6}$$

Hence the distance OP is $(2/\pi)^{1/2}$ and the curve PQR is found by letting t take real values greater than unity giving:

$$\left.\begin{aligned} x &= (2/\pi)^{1/2}\left[1 - (1-t^{-1})\ln(t-1)\right] \\ y &= (2\pi)^{1/2}\,(1-t^{-1}) \end{aligned}\right\} \tag{7}$$

This curve is shown in Fig. 4(a)*

To treat problem (b) we can notice at once that the solution is

$$w = -\left[\zeta^{-1/2} + \zeta^{1/2}\right] \tag{8}$$

since this expression is real on the contour of Fig. 3(b). Integrating to recover the z-plane gives:

$$z = \frac{w^3}{6} - w - \frac{4}{3}\left(\frac{w^2}{4} - 1\right)^{3/2} \tag{9}$$

In this case the distance OP is thus 2/3 and the curve PQRST is given by

*The curve derived from Eq. (7) agrees with the formula of Hurley, loc. cit. However, in a private communication, Hurley has stated that his graph (Fig. 6 in his paper) is in error.

letting w take real values between -2 and 2, and we find

$$\left.\begin{aligned} x &= w^3/6 - w \\ y &= \frac{4}{3}\left(1 - \frac{w^2}{4}\right)^{3/2} \end{aligned}\right\} \tag{10}$$

This curve is shown in Fig. 4(b) and agrees with Cole and Huth (loc. cit.).

We turn now to problem (c). We have not found it possible to find an exact mapping to treat this problem, as shown in Fig. 3(c). Therefore, we have used a modification of the Schwarz-Christoffel transformation due to Cockroft.[5] We define the t-plane by:

$$\frac{d\zeta}{dt} = \frac{-A(t - a)}{t^{1/2}[t^{1/2} + b(t-1)^{1/2}]} \tag{11}$$

The points NOPQR transform to the points $t = -\infty, a(<0), 0, 1, \infty$. Thus the real axis of the t-plane then transforms into the lines NO, OP, a curve joining P and Q, and the line QR. The approximation lies in identifying the curve in the ζ-plane corresponding to the segment PQ in the t-plane with the quadrant of the unit circle. The three constants in (11) are determined by calculating the change in ζ along OP (one real condition) and along PQ (one complex condition). With these values the curve in the ζ-plane corresponding to the segment PQ in the t-plane has been calculated; it is shown in Fig. 5. The maximum deviation from the quadrant of the unit circle is 3%. This corresponds in the physical plane to a magnetic pressure along the segment PQ that exceeds unity by a maximum of 6%. If this approximation be accepted, the problem can be completed by recovering the z-plane as follows: near $\zeta = 0$, $t = a$, (11) gives (since a is negative)

$$\zeta \sim \frac{1/2\, A\, (t - a)^2}{(-a)^{1/2}\,[(-a)^{1/2} + b(1-a)^{1/2}]} \tag{12}$$

The complex potential is therefore

$$w = -c\ (t - a)^{-1} \tag{13}$$

where the constant c is determined from (12), and

$$z = \int \zeta dw = \int_a^t \zeta \frac{dw}{dt}\ dt = \zeta w - \int_a^t w \frac{d\zeta}{dt}\ dt \tag{14}$$

The integrations in Eqs. (11) and (14) are easily performed by choosing $(1 - t^{-1})^{1/2}$ as independent variable. The values of the constants are $a = -2.716$, $b = .4438$, $A = .1601$. The boundary of the magnetic field region is shown in Fig. 4(c).

Conclusion

In Table I we list the principal dimensions of our results, namely, the distance OP (x_P), the co-ordinates of the point Q (x_Q, y_Q) and the y-coordinate of R, y_R, which is asymptotic in character for problems (a) and (c).

Table I

Problem	x_P	x_Q	y_Q	y_R
(a)	.80	1.02	.55	2.51
(b)	.67	.94	.47	1.33
(c)	.69	1.01	.57	2.50

As we anticipated, Problem (c) resembles Problem (b) near the point P and becomes like Problem (a) at R. The agreement between Problems (a) and (c) at Q is, perhaps, more surprising. It seems that, in certain cases at least, one can construct flows having trapped regions in this type of magnetohydrodynamics from simpler situations without fear of substantial error.

Acknowledgment

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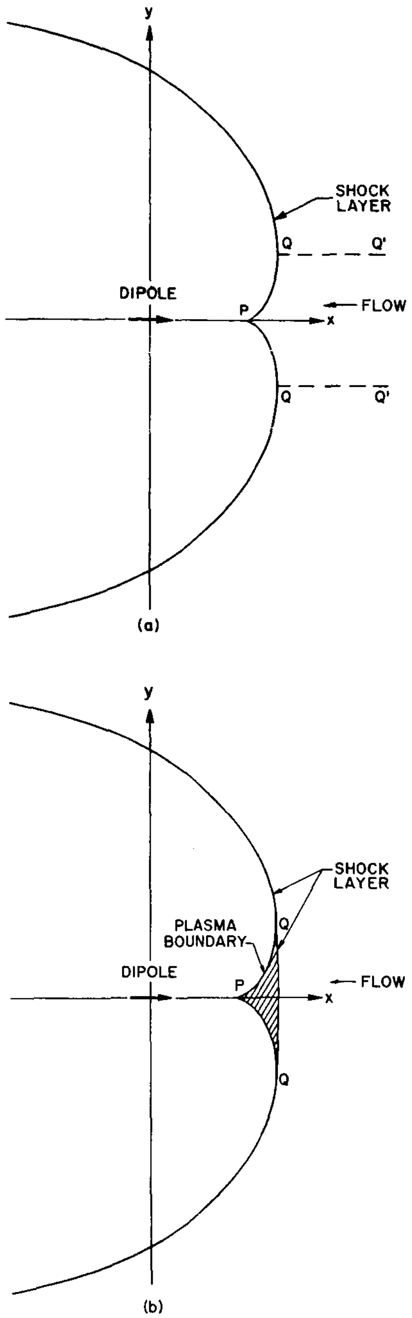

Fig. 1 (a) Re-entrant shape of the cavity formed in a hypersonic stream by a dipole oriented parallel to the flow, when the magnetic pressure is balanced along the boundary by the Newtonian pressure $\rho_\infty u_\infty^2 \sin^2 \alpha$. (b) Proposed form of the flow for this case; the hatched region is stagnant gas at constant pressure $\rho_\infty u_\infty^2$. A nearly normal shock joins the points Q.

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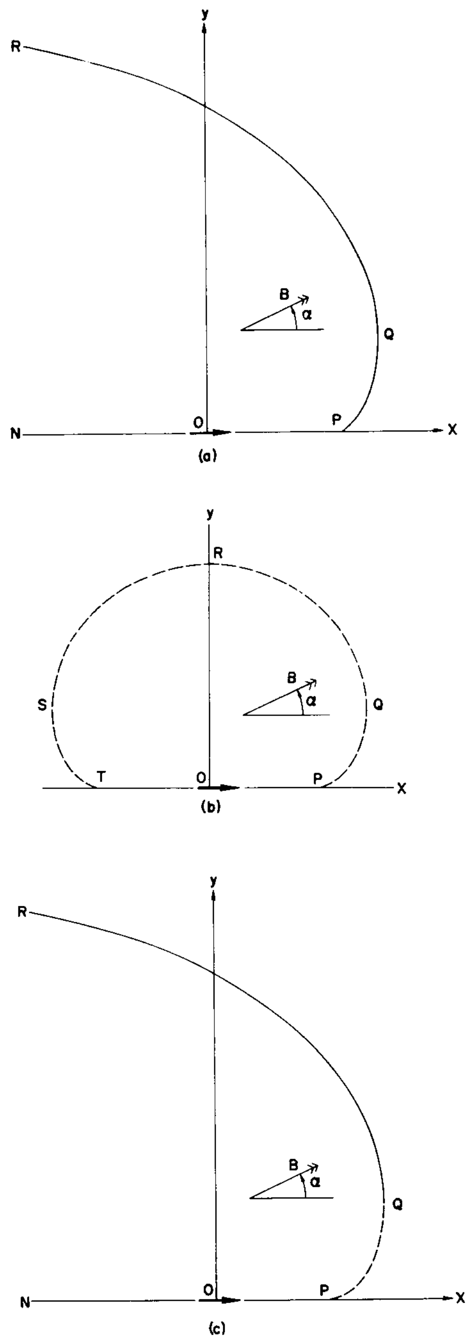

Fig. 2 z-plane for problems (a) (b) and (c). O is the dipole. Solid lines represent portions of the boundary along which the Newtonian pressure condition (1) is applied. Broken lines represent portions of the boundary along which the field has constant (unit) strength.

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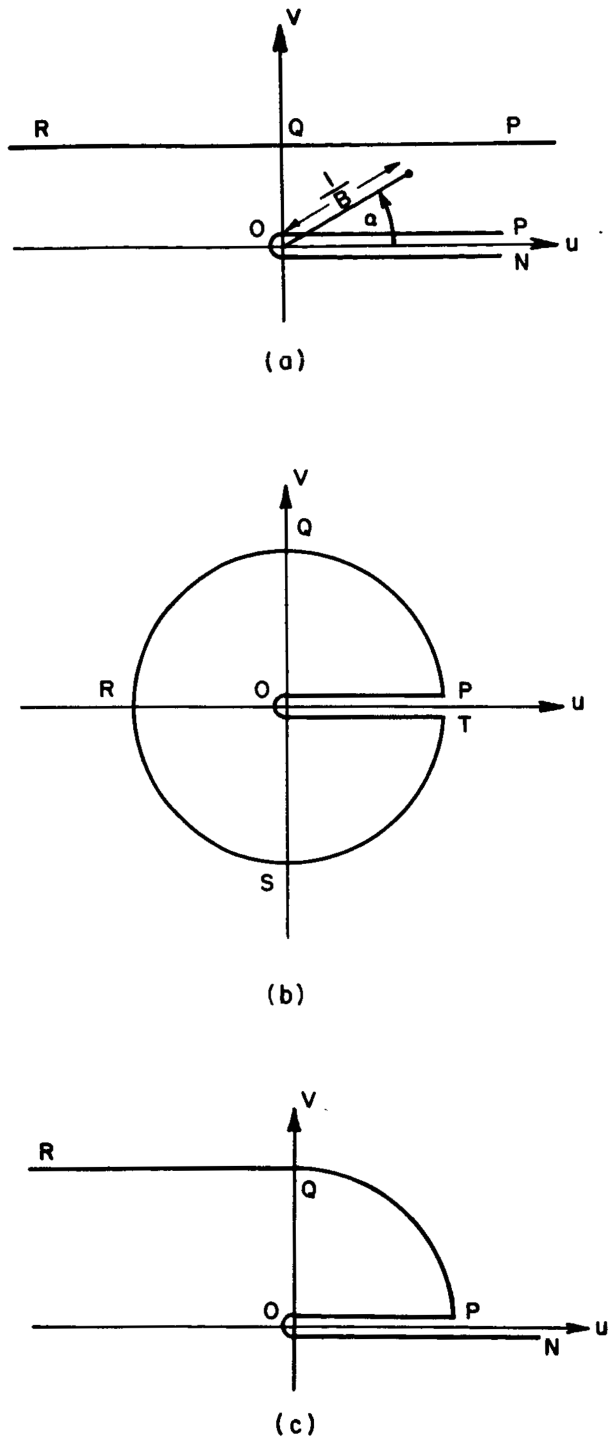

Fig. 3 ζ-plane for problems (a), (b) and (c).

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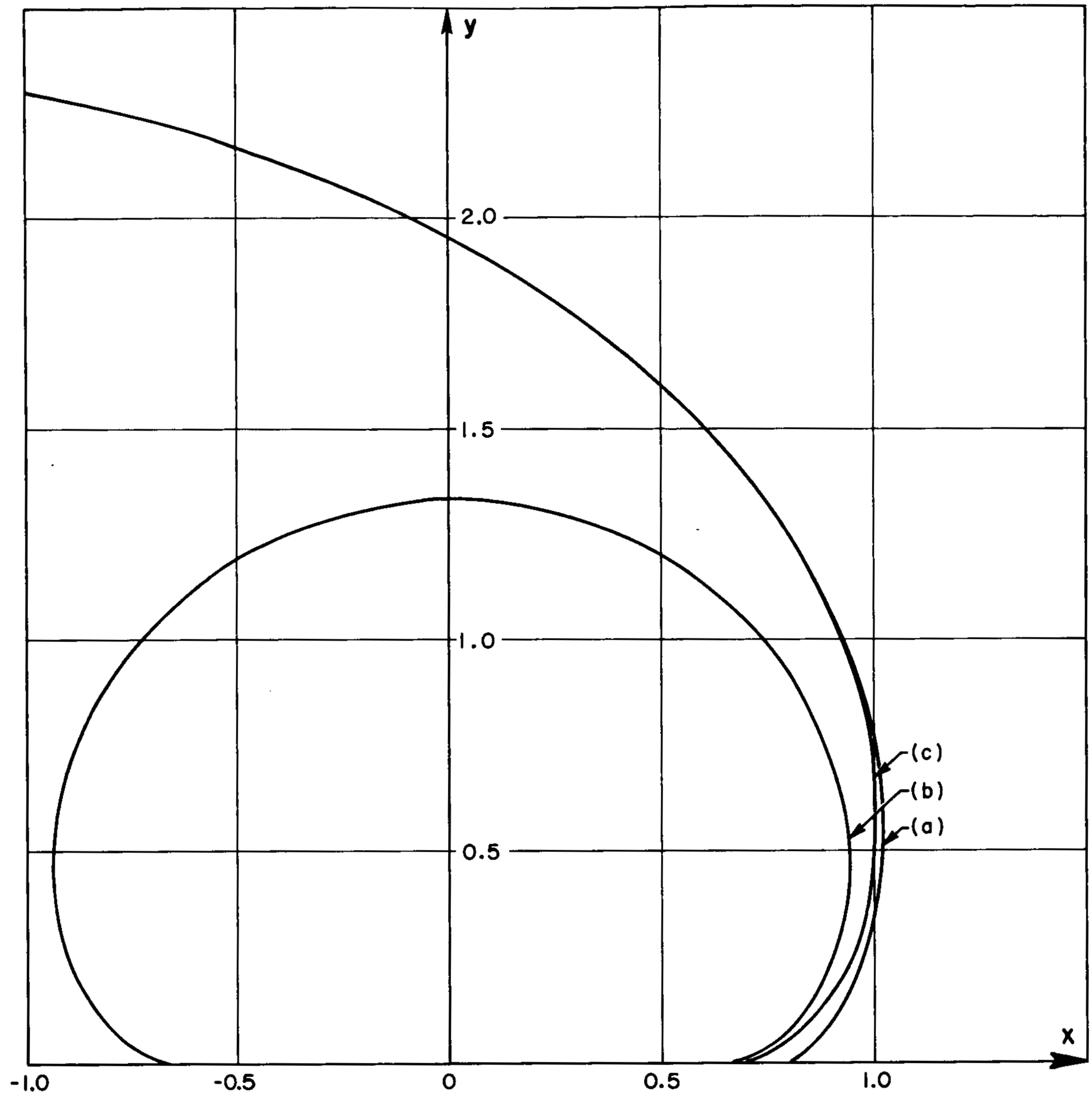

Fig. 4 Boundary shapes for problems (a), (b) and (c).

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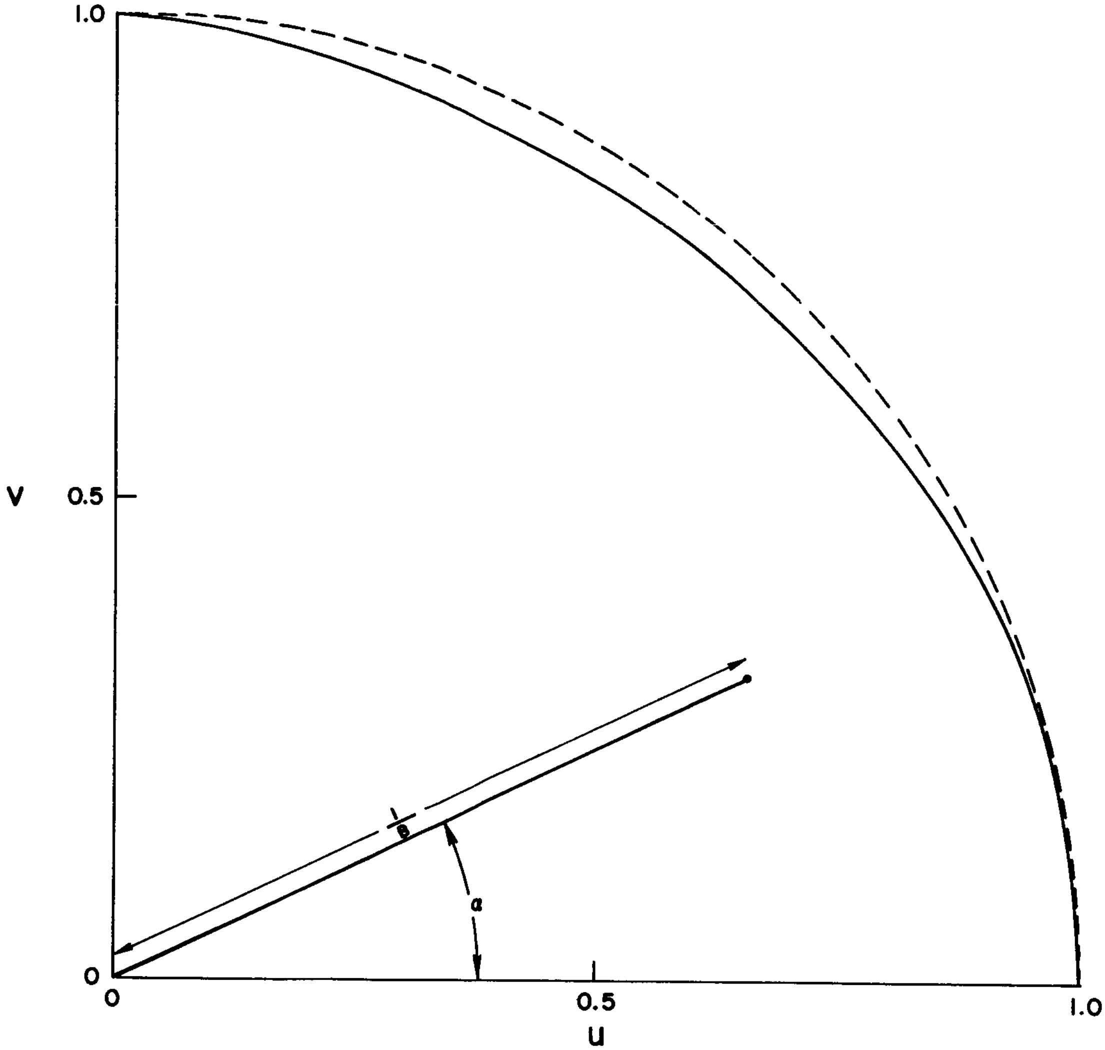

Fig. 5 Part of the ζ-plane for problem (c). The quadrant of the unit circle (broken line) corresponds to Fig. 3(c). The solid line, which is computed from (11), represents our approximation.

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